Wielomiany

Zadanie 1.

Rozwiąż następujące równania:

a) $4x^3-21x^2+29x-6=0$; b) $x^3+5x^2-2x-10=0$; c) $6x^3-19x^2+19x-6=0$;
d) $2x^3-5x^2-5x+2=0$; e) $x^5+2x^4-4x^3-8x^2+3x+6=0$;
f) $x^9-2x^8+3x^5-6x^4+3x-6=0$; g) $9x^4-9x^3+11x^2-9x+2=0$,
h) $8x^3-12x^2-2x+3=0$

Rozwiązanie

a) $4x^3-21x^2+29x-6=0$;

$$4\cdot(-1)^3-21\cdot(-1)^2+29\cdot(-1)-6=-4-21-29-6=-60$$

$$4\cdot 1-21\cdot 1+29\cdot 1-6=4-21+29-6=-2$$

$$4\cdot(-2)^3-21\cdot(-2)^2+29\cdot(-2)-6=4\cdot(-8)-21\cdot 4+29\cdot(-2)-6=$$

$$=-32-84-58-6=-180$$

$$4\cdot 2^3-21\cdot 2^2+29\cdot 2-6=4\cdot 8-21\cdot 4+29\cdot 2-6=32-84+58-6=0$$

$$x_1=2$$

$$4x^3-21x^2+29x-6 : x-2=4x^2-13x+3$$

$$\begin{array}{r} -4x^3+8x^2 \\ \hline -13x^2+29x-6 \\ \underline{13x^2-26x} \\ 3x-6 \\ \underline{-3x+6} \\ = \quad = \end{array}$$

$$4x^2-13x+3=0$$

$$\Delta=169-48=121$$

$$\sqrt{\Delta}=11$$

$$x_2=\frac{13-11}{4\cdot 2}=\frac{1}{4};\qquad x_3=\frac{13+11}{4\cdot 2}=3$$

b) $x^3+5x^2-2x-10=0$

$$(-1)^3+5\cdot(-1)^2-2\cdot(-1)-10=-1+5+2-10=-4$$

$$1^3+5\cdot 1^2-2\cdot 1-10=1+5-2-10=-6$$

$$(-2)^3+5\cdot(2)^2-2\cdot(-2)-10=-8+5\cdot 4+4-10=-8+20+4-10=6$$

$$2^3 + 5 \cdot 2^2 - 2 \cdot 2 - 10 = 8 + 5 \cdot 4 - 4 - 10 = 8 + 20 - 4 - 10 = 14$$

$$(-5)^3 + 5 \cdot (-5)^2 - 2 \cdot (-5) - 10 = -125 + 125 + 10 - 10 = 0$$

$$x_1 = -5$$

$$x^3 + 5x^2 - 2x - 10 : x + 5 = x^2 - 2$$

$$-x^3 - 5x^2$$

$$\underline{\qquad\qquad}$$

$$-2x - 10$$

$$2x + 10$$

$$\underline{\qquad\qquad}$$

$$= \quad =$$

$$x^2 - 2 = 0$$

$$\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) = 0$$

$$x_2 = \sqrt{2}; \qquad x_3 = -\sqrt{2}$$

c) $6x^3 - 19x^2 + 19x - 6 = 0$

$$6 \cdot (-1)^3 - 19 \cdot (-1)^2 + 19 \cdot (-1) - 6 = -6 - 19 - 19 - 6 = -50$$

$$6 \cdot 1^3 - 19 \cdot 1^2 + 19 \cdot 1 - 6 = 6 - 19 + 19 - 6 = 0$$

$$x_1 = 1$$

$$6x^3 - 19x^2 + 19x - 6 : x - 1 = 6x^2 - 13x + 6$$

$$-6x^3 + 6x^2$$

$$\underline{\qquad\qquad}$$

$$-13x^2 + 19x - 6$$

$$13x^2 - 13x$$

$$\underline{\qquad\qquad}$$

$$6x - 6$$

$$-6x + 6$$

$$\underline{\qquad\qquad}$$

$$= \quad =$$

$$6x^2 - 13x + 6 = 0$$

$$\Delta = 169 - 144 = 25$$

$$\sqrt{\Delta} = 5$$

$$x_2 = \frac{13 - 5}{6 \cdot 2} = \frac{8}{12} = \frac{2}{3}; \qquad x_3 = \frac{13 + 5}{6 \cdot 2} = \frac{18}{12} = 1\frac{1}{2}$$

d) $2x^3 - 5x^2 - 5x + 2 = 0$

$$2 \cdot (-1)^3 - 5 \cdot (-1)^2 - 5 \cdot (-1) + 2 = 2 \cdot (-1) - 5 \cdot 1 - 5 \cdot (-1) + 2 = -2 - 5 + 5 + 2 = 0$$

$$x_1 = -1$$

$$2x^3 - 5x^2 - 5x + 2 : x + 1 = 2x^2 - 7x + 2$$

$$\begin{array}{r} -2x^3 - 2x^2 \\ \hline -7x^2 - 5x + 2 \\ 7x^2 + 7x \\ \hline 2x + 2 \\ -2x - 2 \\ \hline = \quad = \end{array}$$

$$2x^2 - 7x + 2 = 0$$

$$\Delta = 49 - 16 = 33$$

$$\sqrt{\Delta} = \sqrt{33}$$

$$x_2 = \frac{7 - \sqrt{33}}{2 \cdot 2} = \frac{7 - \sqrt{33}}{4}; \qquad x_3 = \frac{7 + \sqrt{33}}{2 \cdot 2} = \frac{7 + \sqrt{33}}{4}$$

e) $x^5 + 2x^4 - 4x^3 - 8x^2 + 3x + 6 = 0$

$$(-1)^5 + 2 \cdot (-1)^4 - 4 \cdot (-1)^3 - 8 \cdot (-1)^2 + 3 \cdot (-1) + 6 =$$

$$= -1 + 2 + 4 - 8 - 3 + 6 = 0$$

$$x_1 = -1$$

$$x^5 + 2x^4 - 4x^3 - 8x^2 + 3x + 6 : x + 1 = x^4 + x^3 - 5x^2 - 3x + 6$$

$$\begin{array}{r} -x^5 - x^4 \\ \hline x^4 - 4x^3 - 8x^2 + 3x + 6 \\ -x^4 - x^3 \\ \hline -5x^3 - 8x^2 + 3x + 6 \\ 5x^3 + 5x^2 \\ \hline -3x^2 + 3x + 6 \\ 3x^2 + 3x \\ \hline 6x + 6 \\ -6x - 6 \\ \hline = \quad = \end{array}$$

$$x^4 + x^3 - 5x^2 - 3x + 6 = 0$$

$$(-1)^4 + (-1)^3 - 5 \cdot (-1)^2 - 3 \cdot (-1) + 6 = 1 - 1 - 5 + 3 + 6 = 4$$

$$1^4 + 1^3 - 5 \cdot 1^2 - 3 \cdot 1 + 6 = 1 + 1 - 5 - 3 + 6 = 0$$

$$x_2 = 1$$

$$x^4 + x^3 - 5x^2 - 3x + 6 : x - 1 = x^3 + 2x^2 - 3x - 6$$

$$-x^4 + x^3$$

$$2x^3 - 5x^2 - 3x + 6$$

$$-2x^3 + 2x^2$$

$$-3x^2 - 3x + 6$$

$$3x^2 - 3x$$

$$-6x + 6$$

$$6x - 6$$

$$= \quad =$$

$$x^3 + 2x^2 - 3x - 6 = 0$$

$$1^3 + 2 \cdot 1^2 - 3 \cdot 1 - 6 = 1 + 2 - 3 - 6 = -6$$

$$(-2)^3 + 2 \cdot (-2)^2 - 3 \cdot (-2) - 6 = -8 + 2 \cdot 4 + 6 - 6 = -8 + 8 + 6 - 6 = 0$$

$$x_3 = -2$$

$$x^3 + 2x^2 - 3x - 6 : x + 2 = x^2 - 3$$

$$-x^3 - 2x^2$$

$$-3x - 6$$

$$3x + 6$$

$$= \quad =$$

$$x^2 - 3 = 0$$

$$\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) = 0$$

$$x_4 = \sqrt{3}; \qquad x_5 = -\sqrt{3}$$

f) $x^9 - 2x^8 + 3x^5 - 6x^4 + 3x - 6 = 0$

$$(-1)^9 - 2 \cdot (-1)^8 + 3 \cdot (-1)^5 - 6 \cdot (-1)^4 + 3 \cdot (-1) - 6 = -1 - 2 - 3 - 6 - 3 - 6 = -21$$

$$1^9 - 2 \cdot 1^8 + 3 \cdot 1^5 - 6 \cdot 1^4 + 3 \cdot 1 - 6 = 1 - 2 + 3 - 6 + 3 - 6 = -7$$

$$(-2)^9 - 2 \cdot (-2)^8 + 3 \cdot (-2)^5 - 6 \cdot (-2)^4 + 3 \cdot (-2) - 6 = -512 - 512 - 96 - 96 - 6 - 6 = -1228$$

$$2^9 - 2 \cdot 2^8 + 3 \cdot 2^5 - 6 \cdot 2^4 + 3 \cdot 2 - 6 = 512 - 512 + 96 - 96 + 6 - 6 = 0$$

$$x_1 = 2$$

$$x^9 - 2x^8 + 3x^5 - 6x^4 + 3x - 6 = 0 : x - 2 = x^8 + 3x^4 + 3$$

$$\underline{-x^9 + 2x^8}$$

$$3x^5 - 6x^4 + 3x - 6$$

$$\underline{-3x^5 + 6x^4}$$

$$3x - 6$$

$$\underline{-3x + 6}$$

$$= \quad =$$

Ponieważ $x^8 + 3x^4 + 3 \geq 3$ więc równanie nie ma więcej pierwiastków.

g) $9x^4 - 9x^3 + 11x^2 - 9x + 2 = 0$

$$9 \cdot (-1)^4 - 9 \cdot (-1)^3 + 11 \cdot (-1)^2 - 9 \cdot (-1) + 2 = 9 + 9 + 11 + 9 + 2 = 40$$

$$9 \cdot 1^4 - 9 \cdot 1^3 + 11 \cdot 1^2 - 9 \cdot 1 + 2 = 4$$

$$9 \cdot (-2)^4 - 9 \cdot (-2)^3 + 11 \cdot (-2)^2 - 9 \cdot (-2) + 2 = 144 + 72 + 44 + 18 + 2 = 280$$

$$9 \cdot 2^4 - 9 \cdot 2^3 + 11 \cdot 2^2 - 9 \cdot 2 + 2 = 144 - 72 + 44 - 18 + 2 = 100$$

$$9 \cdot \left(\frac{1}{3}\right)^4 - 9 \cdot \left(\frac{1}{3}\right)^3 + 11 \cdot \left(\frac{1}{3}\right)^2 - 9 \cdot \frac{1}{3} + 2 = \frac{9}{81} - \frac{9}{27} + \frac{11}{9} - 3 + 2 =$$

$$= \frac{1}{9} - \frac{1}{3} + 1\frac{2}{9} - 3 + 2 = 1\frac{1}{3} - \frac{1}{3} - 1 = 0$$

$$x_1 = \frac{1}{3}$$

$$9x^4 - 9x^3 + 11x^2 - 9x + 2 : x - \frac{1}{3} = 9x^3 - 6x^2 + 9x - 6$$

$$\underline{-9x^4 + 3x^3}$$

$$\underline{-6x^3} + 11x^2 - 9x + 2$$

$$\underline{6x^3 - 2x^2}$$

$$9x^2 - 9x + 2$$

$$\underline{-9x^2 + 3x}$$

$$\underline{-6x +} 2$$

$$6x - 2$$

$$\underline{= \quad} =$$

$$9x^3 - 6x^2 + 9x - 6 = 0$$

$$3x^3 - 2x^2 + 3x - 2 = 0$$

$$x^2(3x - 2) + 3x - 2 = 0$$

$$(3x - 2)(x^2 + 1) = 0$$

$$3x - 2 = 0$$

$$3x = 2$$

$$x = \frac{2}{3}$$

h) $8x^3 - 12x^2 - 2x + 3 = 0$

$$4x^2 \cdot (2x - 3) - (2x - 3) = 0$$

$$(2x - 3)(4x^2 - 1) = 0$$

$$(2x - 3)(2x - 1)(2x + 1) = 0$$

$$x = \frac{3}{2} \quad lub \quad x = \frac{1}{2} \quad lub \quad x = -\frac{1}{2}$$

Zadanie 2.

Długości boków w pewnym trójkącie stanowią ciąg arytmetyczny o długości 1. Wyznacz długości boków tego trójkąta wiedząc, że jego pole wynosi $\frac{3}{4}\sqrt{15}$.

Rozwiązanie

Niech średni bok trójkąta ma długość x. Wówczas najkrutszy bok ma x – 1 , a najdłuższy x +1.

Obwód tego trójkąta wynosi

$$Ob = 3x$$

Połowa obwodu wynosi

$$p = \frac{3}{2}x$$

Zastosujemy wzór Herona na pole trójkąta

$$P = \sqrt{p \cdot (p-a) \cdot (p-b) \cdot (p-c)} = \sqrt{\frac{3}{2}x \cdot \left(\frac{3}{2}x - x + 1\right) \cdot \left(\frac{3}{2}x - x\right) \cdot \left(\frac{3}{2}x - x - 1\right)} =$$

$$= \sqrt{\frac{3}{2}x \cdot \left(\frac{1}{2}x + 1\right) \cdot \frac{1}{2}x \cdot \left(\frac{1}{2}x - 1\right)} = \sqrt{\frac{3}{4}x^2 \cdot \left(\frac{1}{4}x^2 - 1\right)}$$

Z treści zadania wynika, że

$$\sqrt{\frac{3}{4}x^2 \cdot \left(\frac{1}{4}x^2 - 1\right)} = \frac{3}{4}\sqrt{15}$$

Po podniesieniu obu stron do kwadratu mamy

$$\frac{3}{4}x^2 \cdot \left(\frac{1}{4}x^2 - 1\right) = \frac{9}{16} \cdot 15$$

$$x^2 \cdot \left(\frac{1}{4}x^2 - 1\right) = \frac{45}{4}$$

$$\frac{1}{4}x^4 - x^2 - \frac{45}{4} = 0$$

$$x^4 - 4x^2 - 45 = 0$$

Mamy do rozwiązania równanie dwukwadratowe

Niech

$$x^2 = y$$

Wówczas

$$y^2 - 4y - 45 = 0$$

$$\Delta = 16 + 180 = 196$$

$$\sqrt{\Delta} = 14$$

$$y_1 = \frac{4 - 14}{2} = -5 \quad lub \quad y_2 = \frac{4 + 14}{2} = 9$$

Ponieważ y musi być dodatnie więc $y = 9$

Podobnie x jest dodatnie więc $x = 3$.

Trójkąt ma boki o długościach: 2; 3; 4